# COMPUTATIONAL MECHANICS

## *An Ideal Research Area for Innovative Solutions of Challenging Engineering Problems*


Herbert A. Mang

Institute for Mechanics of Materials and Structures,
Vienna University of Technology
*National RPGE Chair Professor, Tongji University, Shanghai, China

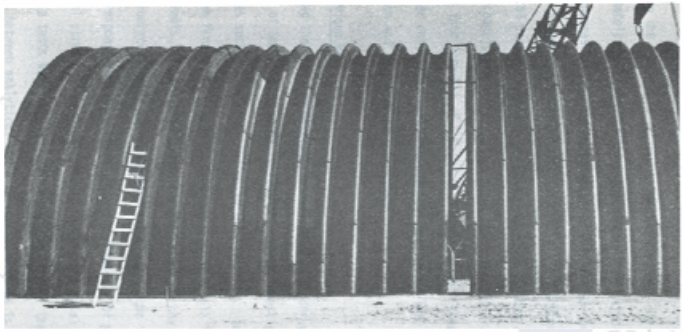
# Prolog

- The advent of the digital computer and the parallel development of the FEM and the BEM have paved the way to challenging industrial applications of nonlinear mechanics.
- Computational (nonlinear) mechanics has become a scientific spearhead of technological progress.
- Computational mechanics is firmly embedded in the computational sciences, including computational mathematics, physics, chemistry, biology, etc. → this is the consequence of the increasing awareness of the importance of a holistic approach in the engineering sciences.
- The trend to consider information from small scales for determination of material properties of heterogeneous materials has stimulated multiscale analysis, which would be impossible without nonlinear computational mechanics.
- Having been involved in the development of computational mechanics for nearly 50 years, my talk has an autobiographical touch related to the topic of the lecture.

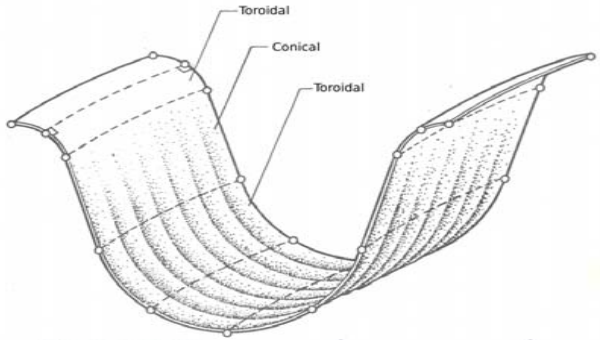
# Doubly corrugated shells – Example for dual use of a structure (1)

1972-1974: Doctoral dissertation (Ph.D.) at Texas Tech

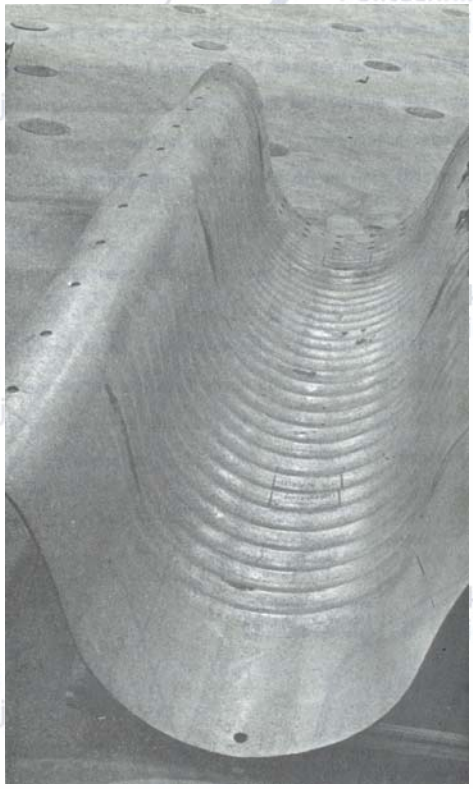
Topic: Finite Element (FE) Analysis of Doubly Corrugated Shells



*Doubly Corrugated Shell*



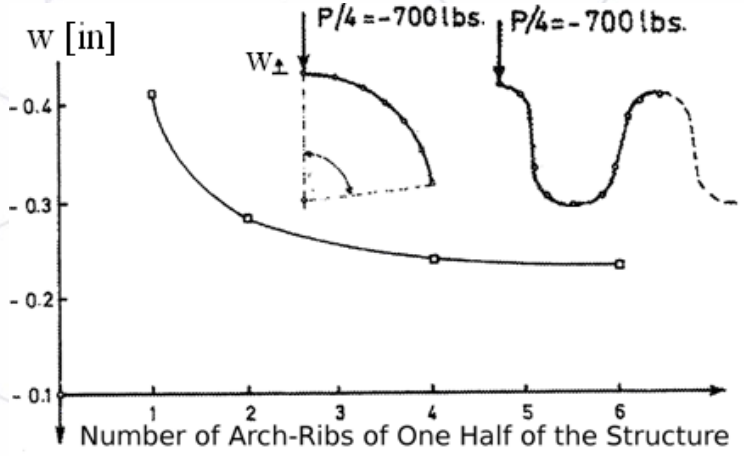

*Finite Element Model of a Portion of the Panel*



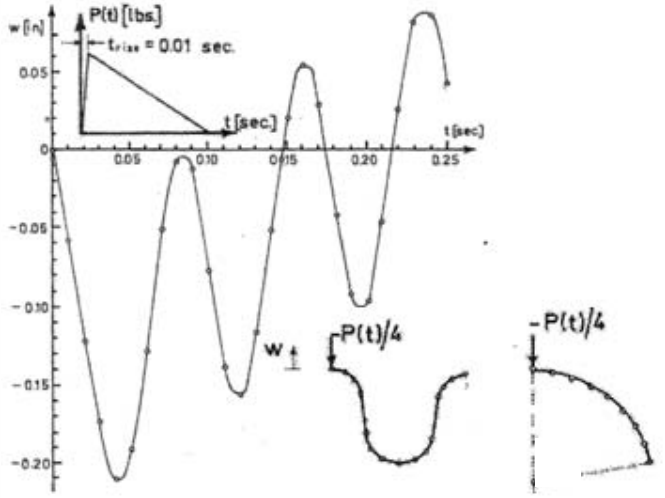
Basis of FE analysis:

Linear strain-displacement equations by Sanders. Difference from the respective equations by Novoshilov only affects (mixed component of bending strains)

# Doubly corrugated shells – Example for dual use of a structure (2)

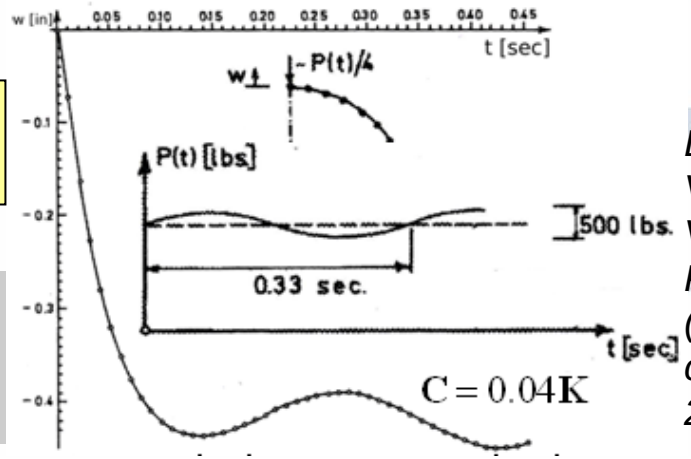



*Shell Effect (static analysis)*




*Undamped Vibrations; Vertical Pulse (structure consisting of 2 panels)*

Numerical integration for dynamic analysis based on Newmark's method



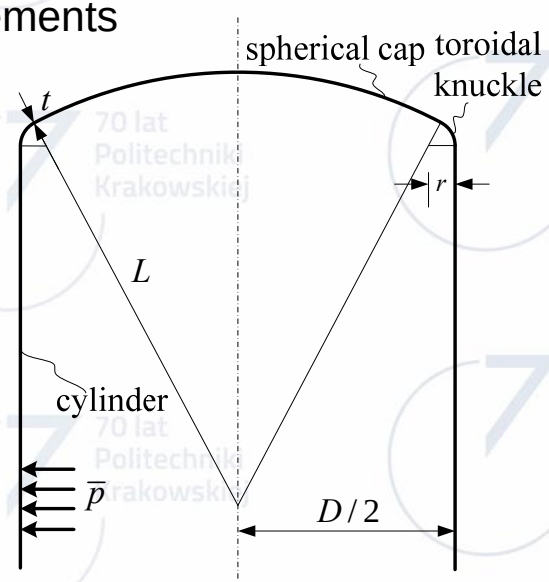

*Damped Vibrations; Vertical Pulse (structure consisting of 2 panels)*

Publication: H.A. Mang, C.V.G. Girÿa Vallabhan, Jimmy H. Smith, *Finite Element Analysis of Doubly Corrugated Shells*. American Society of Civil Engineers. Journal of the Structural Division 102 (1976), pp: 2033-2051.

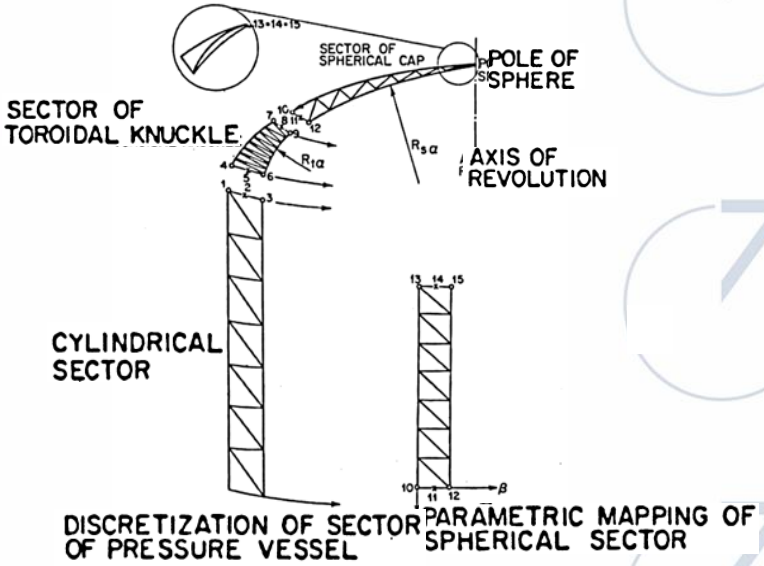
# Computational structural stability analysis (1)

1975-1976: Max-Kade Fellow at Cornell University, writing of habilitation thesis.

One of several additional scientific activities during the stay at Cornell University was instability analysis of torispherical pressure vessel heads with triangular thin-shell finite elements




*Geometry of Torispherical Pressure Vessel Head*

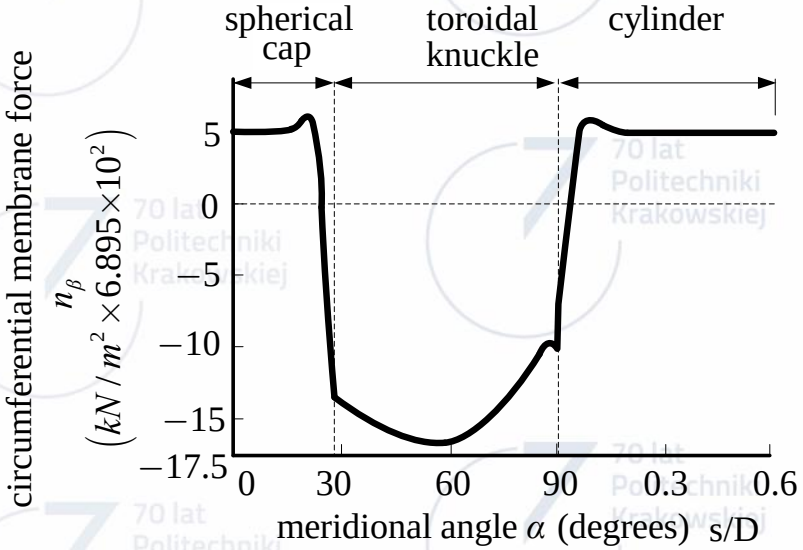



*Discretization of Sector of Pressure Vessel and Parametric Mapping of Spherical Sector*

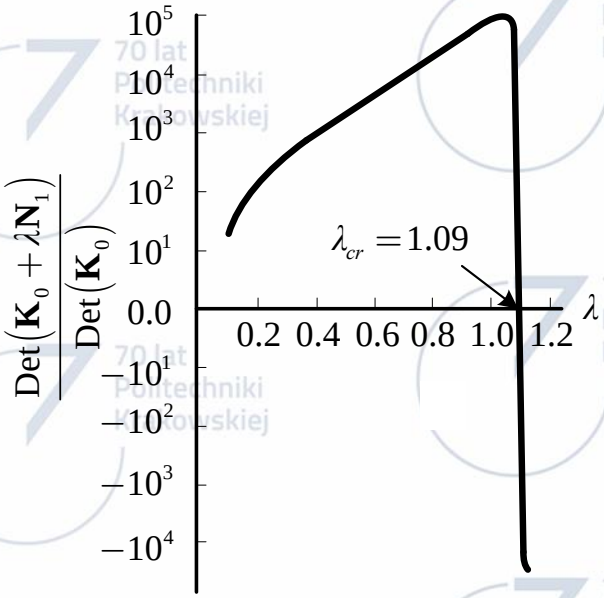
Buckling under external pressure is a well-known possibility. What was less well known at the time of performing this investigation is that buckling may also occur under internal pressure.

# Computational structural stability analysis (2)

Instability analysis of torispherical pressure vessel head under internal pressure




*Circumferential Membrane Force $n_\beta$ for Torispherical Pressure Vessel Head Under Internal Pressure*



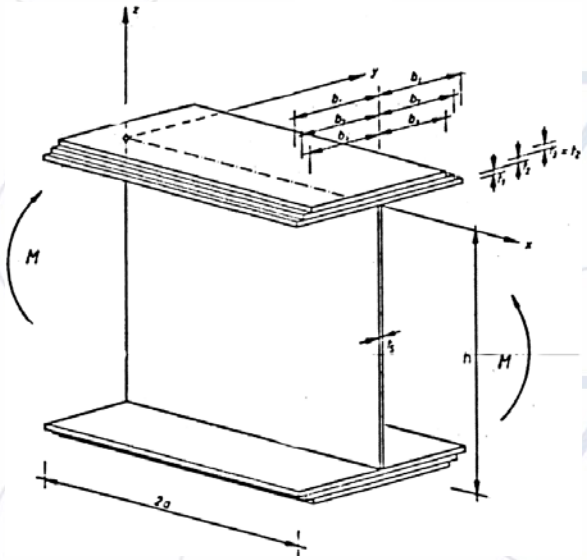

*Normalized Determinant Versus Internal Pressure for Torispherical Pressure Vessel Head*

Publication: V.L. Kanodia, H.A. Mang, R.H. Gallagher, *Instability Analysis of Torispherical Pressure Vessel Heads with Triangular Thin-Shell Finite Elements.* American Society of Mechanical Engineers. Journal of Pressure Vessel Technology 99 (1977), pp.103-113.

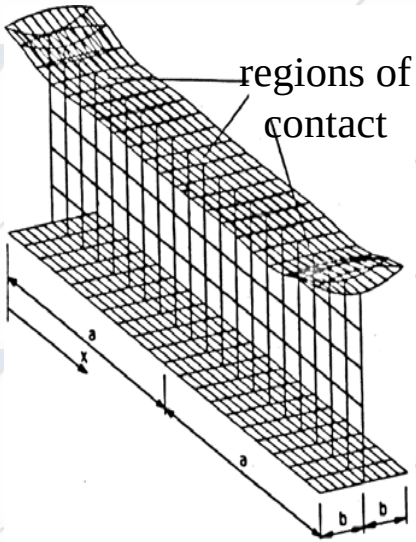
Open Question:
What is the physical meaning of the maximum of the determinant?

# Computational structural stability analysis (3)

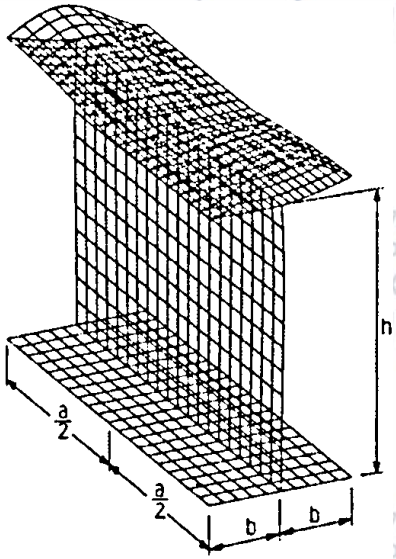
Buckling of Multi-Lamellae Compression Flanges of Welded I-Beams:
A Unilateral Elastic-Plastic Plate-Stability Problem




*Section of an I-beam with Flanges Consisting of Three Lamellae*




*Wave of a Symmetric Periodic Buckling Mode*

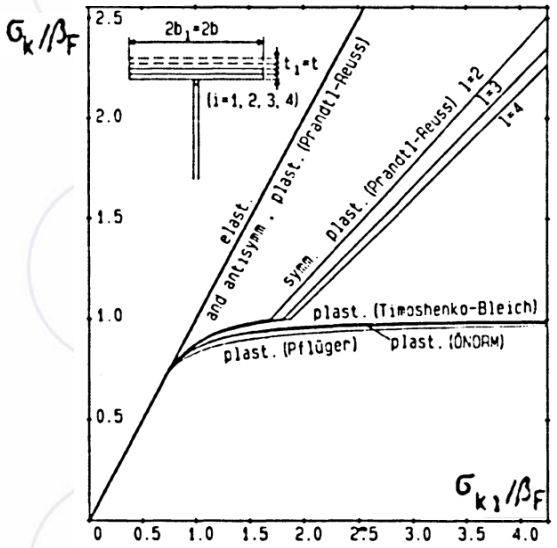



*Representative of One Out of Two Categories of Unsymmetric Eigenforms*

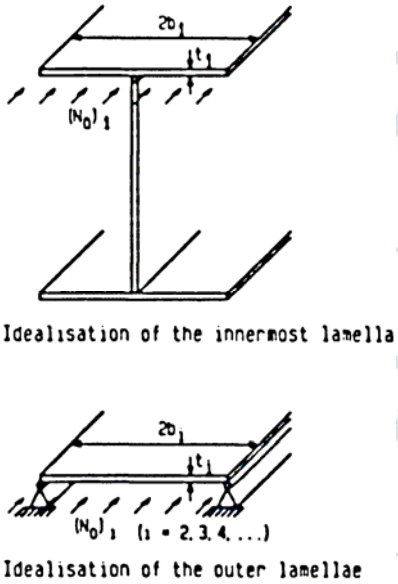
Publication: Z.S. Chen, H. A. Mang, *Buckling of Multi-Lamellae Compression Flanges of Welded I-Beams: A Unilateral Elasto-Plastic Plate-Stability Problem.* International Journal for Numerical Methods in Engineering 26 (1988) 1403-1441.

# Computational structural stability analysis (4)

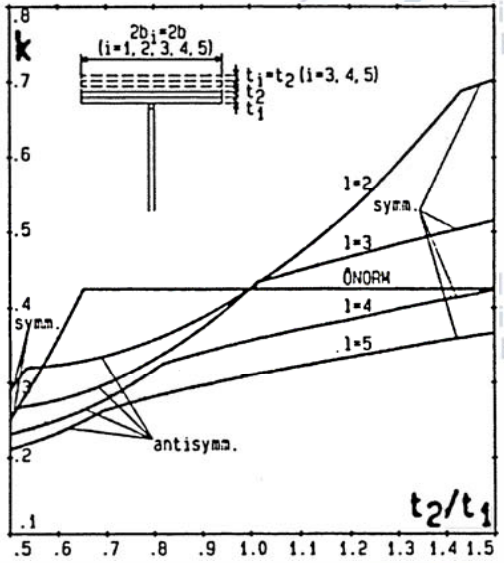
## Buckling of Multi-Lamellae Compression Flanges of Welded I-Beams:




$\sigma_k/\beta_F = f(\sigma_{kl}/\beta_F)$ *for Symmetric and Antisymmetric Buckling of Compression Flanges Consisting of Two to Five Lamellae of Equal Thickness*




*Classical Design Procedure for Multi-Lamellae Compression Flanges of Welded I-Beams*




*Comparison of* $k = Min(k_{sym}, k_{antim}) = f(t_2, t_1)$, $(l = 2, 3, 4,)$, *Based on the Present Investigation with* $k = t_2/t_1$ *Resulting from the Classical Design Procedure*

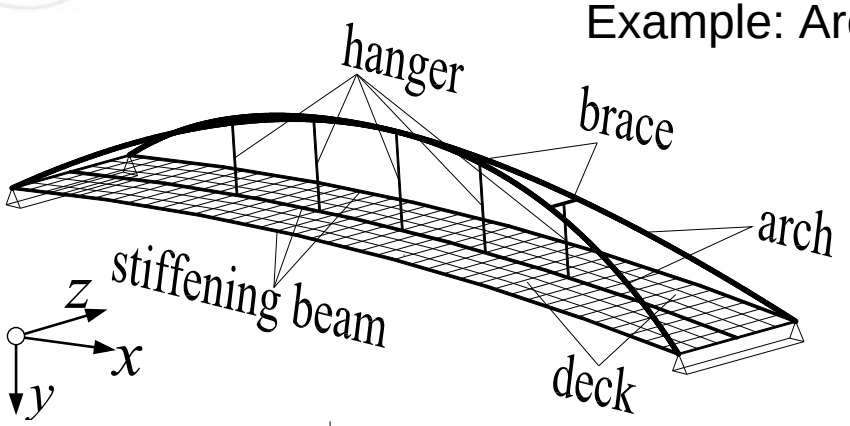
The curves based on the Pflüger flow rule and the diagrams based on the Timoshenko-Bleich constitutive model are identical for symmetric and antisymmetric buckling, and they do not depend on the number of lamellae.

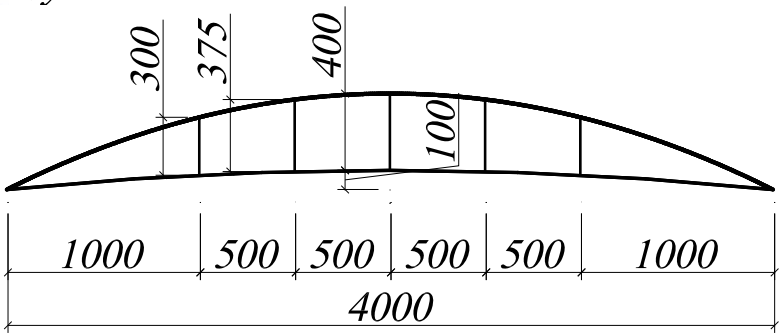
The latter may be on the safe or on the unsafe side

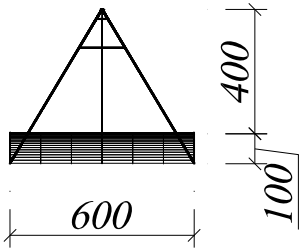
# Computational structural stability analysis (5)

Conversion of imperfection-sensitive elastic structures into imperfection-insensitive ones by modifications of the original design

Example: Arch bridge




p = 0.04 kN/cm²
(applied on the deck surface including self weight and traffic load)

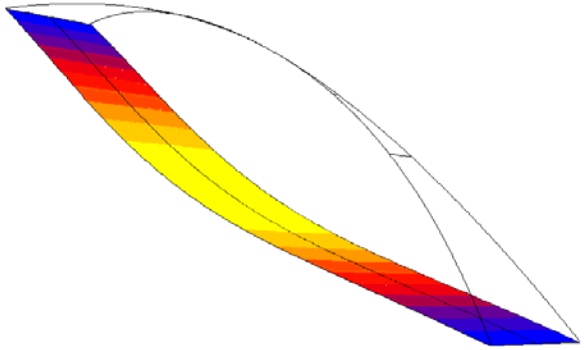
units in [cm]

Publication: X. Jia, H.A. Mang, *Conversion of Imperfection-Sensitive Elastic Structures into Imperfection Insensitive Ones by Adding Tensile Members. Journal of the International Association for Shell and Spatial Structures 52 (2011) 121-128.*

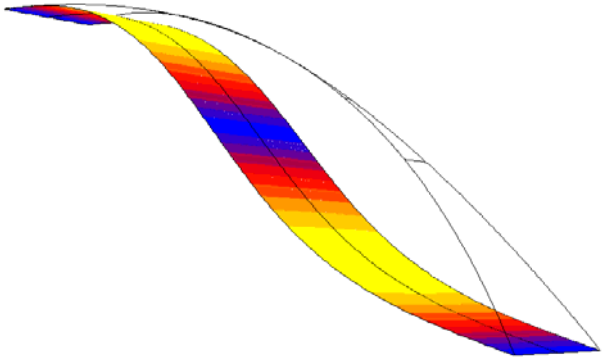
# Computational structural stability analysis (6)

Example: Arch bridge

**Results for the arch bridge with $\kappa = 0$ (no hangers)**



*Deformed Arch Bridge Just Before Buckling*



*Buckling Mode*

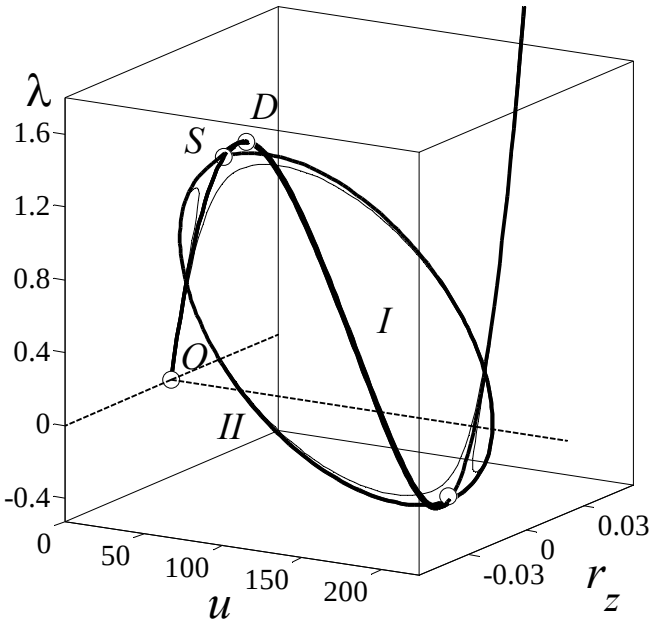
For the reference load, the vertical displacement of the midpoint of the arch bridge is *21.1cm*, which is *1/189* of the span.

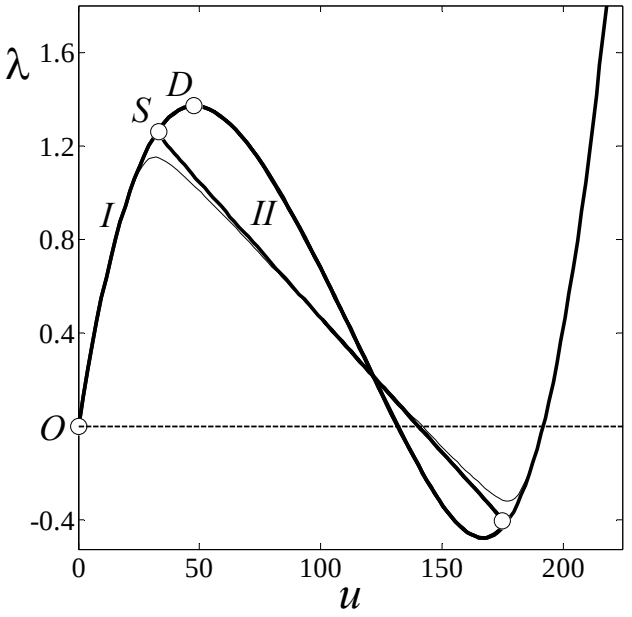
At the onset of buckling, the deck is mainly in compression. Hence, the influence of the reinforcement ratio and of cracking of concrete on the buckling load and the initial postbuckling behavior is negligible.

In the prebuckling domain, the stress state consists of membrane and bending stresses and transverse shear stresses.

# Computational structural stability analysis (7)

Example: Arch bridge with $\kappa = 0$ (no hangers)



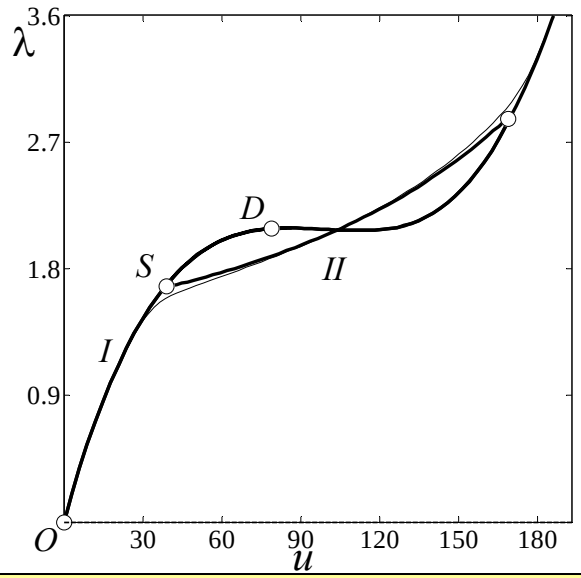




$\Rightarrow$ Symmetric bifurcation $\Rightarrow$ $\lambda_1 = 0$

$\Rightarrow$ Negative slope at bifurcation point $\Rightarrow$ $\lambda_2 < 0$ $\Rightarrow$ imperfection sensitive

# Computational structural stability analysis (8)

Example: **Arch bridge with $\kappa = 0.8000$ (with hangers)**




$\Rightarrow$ $\lambda_2 > 0 \Rightarrow$ The structure is imperfection insensitive.

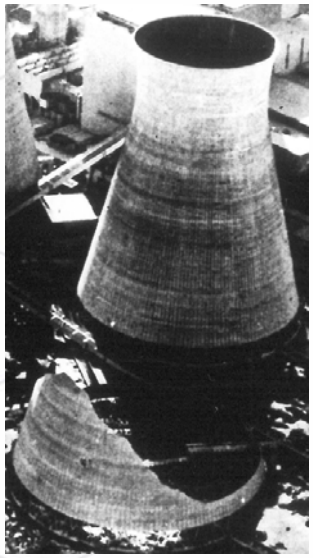
$\Rightarrow$ The transition from the imperfection-sensitive arch bridge into an insensitive one is the consequence of adding tensile members.

$\Rightarrow$ Moreover, the tensile members result in an increase of the stability limit.

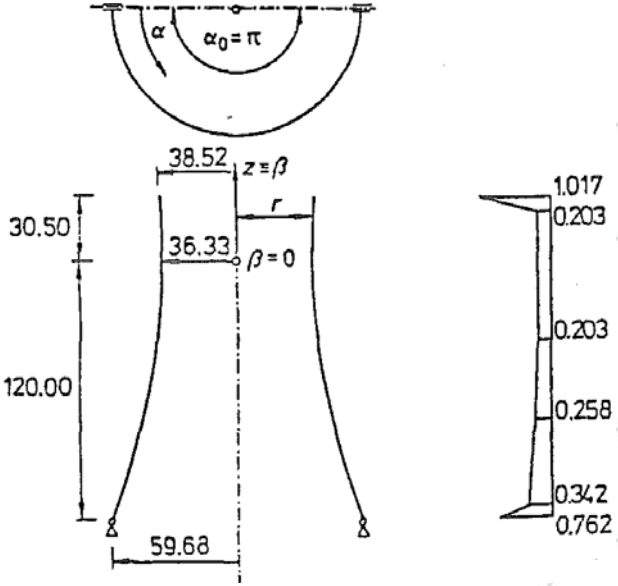
$\Rightarrow$ For the reference load, the vertical displacement of the midpoint of the arch bridge is 12.9cm, which is *1/310* of the span.

# Computational mechanics of reinforced concrete structures (1)

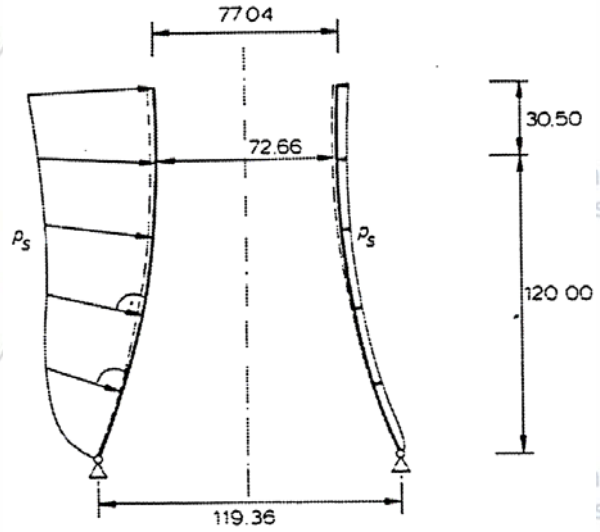
Practical research topic in the area of computational mechanics of plates and shells made of reinforced concrete:
Wind-loaded reinforced-concrete cooling towers: buckling or ultimate load?



*Collapse of Cooling Towers in Ferrybridge, UK, in 1965*




*Cooling Tower at Port Gibson, Miss., USA. Characteristic Dimensions*

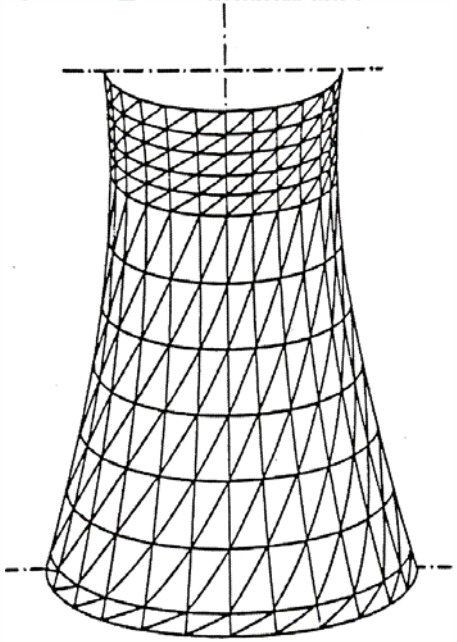



*Wind Profile for Luff and Lee Meridian*

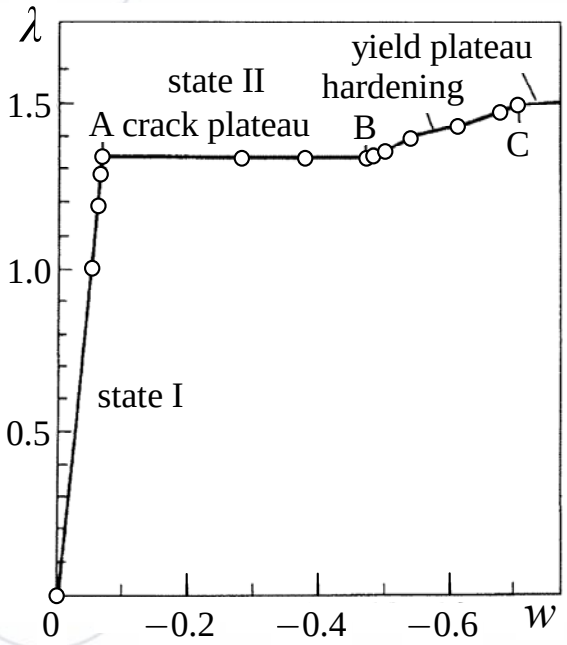
Previously, the school of thought in engineering was that wind-loaded hyperboloid cooling towers made of reinforced concrete fail by progressive damage and loss of material strength rather than buckling

# Computational mechanics of reinforced concrete structures (2)

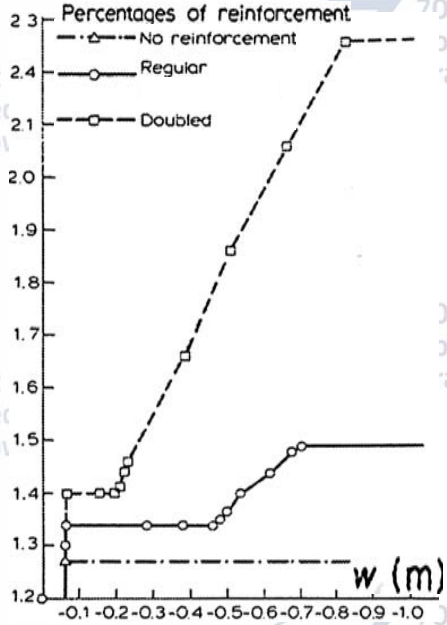
Wind-loaded reinforced-concrete cooling towers: buckling or loss of material strength?



*Cooling Tower at Port Gibson, Miss., USA, Finite Element Mesh*




*Transverse Displacement at the Point of Intersection of the Luff Meridian and the Throat of the Shell Versus the Load Factor*

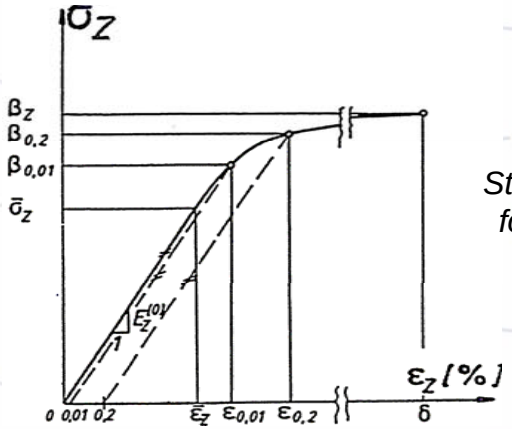



*Transverse Displacement w at the Mentioned Point Versus the Load Factor for Three Different Percentages of Reinforcement*

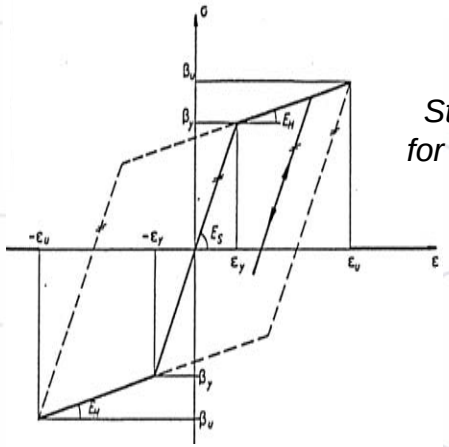
Conclusion: Buckling loads from linear as well as geometrically nonlinear analyses are considerably larger than the ultimate load $\Rightarrow$ failure by loss of strength

# Computational mechanics of reinforced concrete structures (3)

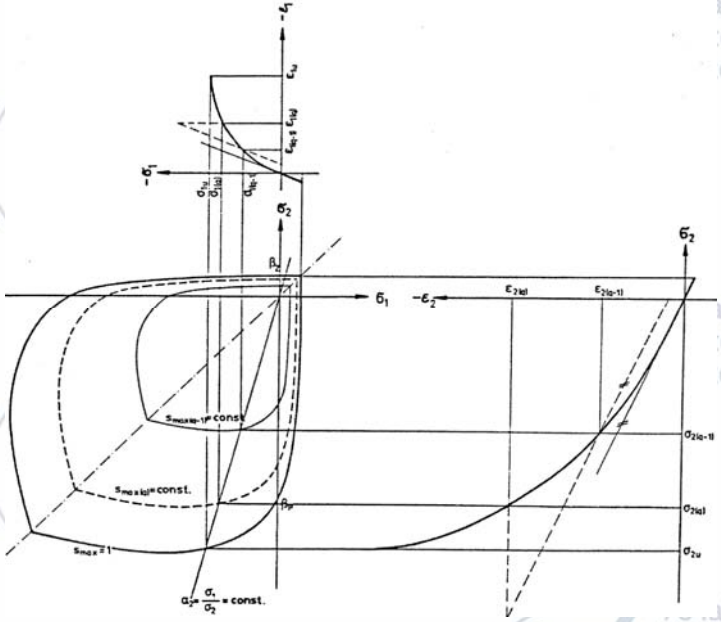
## Long-time Deformations of Prestressed Concrete Shells




*Stress-strain Diagram for the Prestressing Steel*




*Stress-strain Diagram for the Reinforcing Steel*

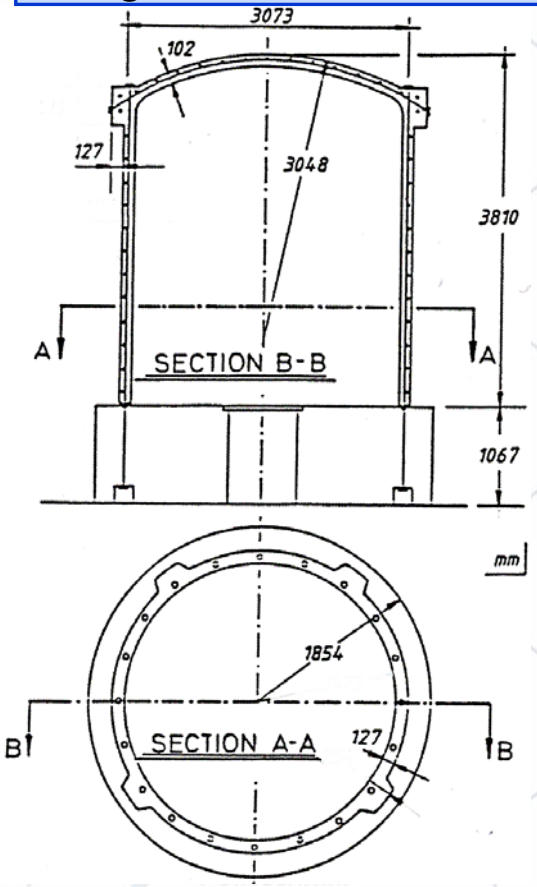


*Loading Curves, Fracture Envelope and Stress-strain Diagrams for a Specific Ratio of*

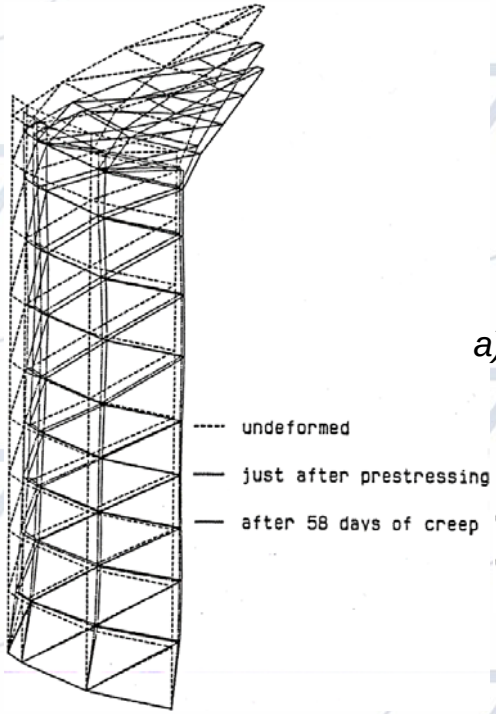
Publication: H. Walter, G. Hofstetter, H.A. Mang, *Long-Time Deformations and Creep Buckling of Prestressed Concrete Shells. Computational Mechanics of Nonlinear Response of Shells.* Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1990, 378-405.

# Computational mechanics of reinforced concrete structures (4)

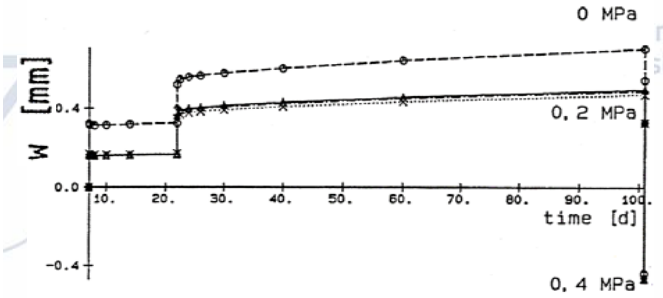
## Long-time Deformations of Prestressed Concrete Shells: Numerical Investigation



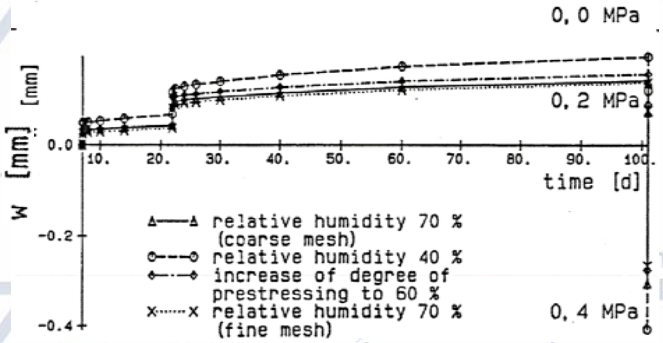

*Model of a Prestressed Reactor Secondary Confinement*




*State of Displacements*
*a)Just After the Application of Prestress*
*b)After 58 Days of Creep (300-fold Superelevated)*




*a) Midheight of the Cylindrical Part of the Shell*

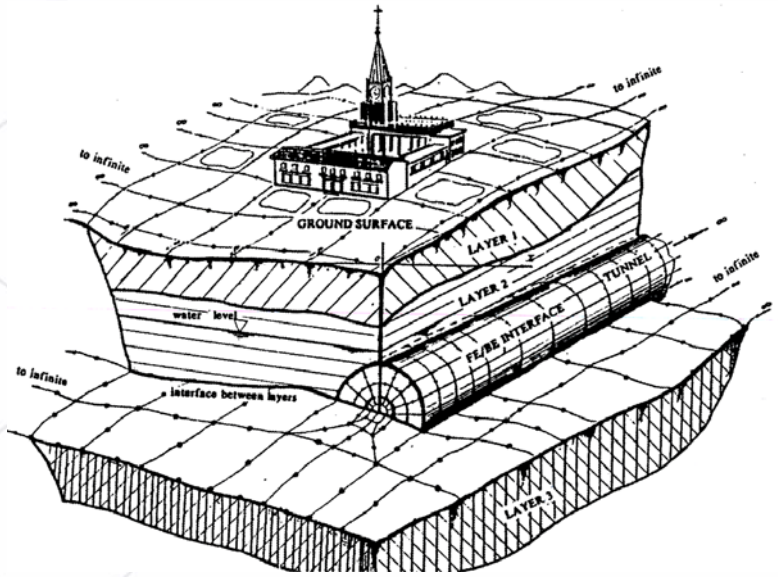



*b) Pole of the Spherical Cap*

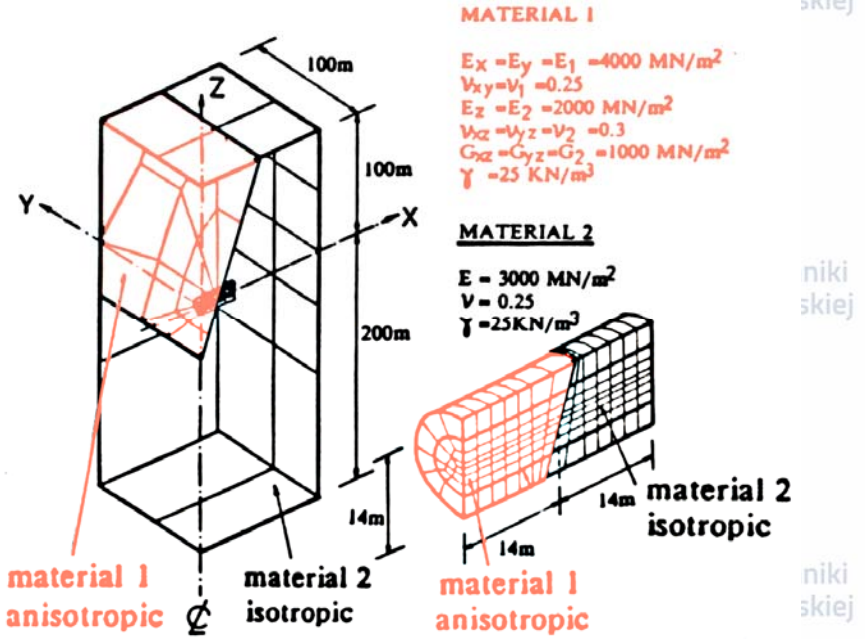
*Time-displacement Paths for two Points of the Structure*

# Computational mechanics in tunnel construction (1)

## Coupling of FE-and BE-Discretizations for 3D-Stress Analysis of Tunnels in Anisotropic Rock




*Coupled FE and BE Discretizations for 3D-Stress Analysis of a Tunnel*

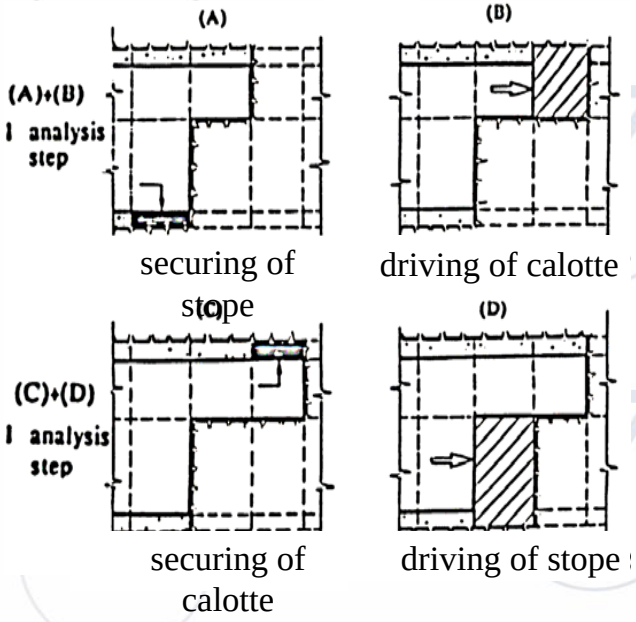



*BE-FE Model for 3D-Stress Analysis of a Stretch of a Tunnel*

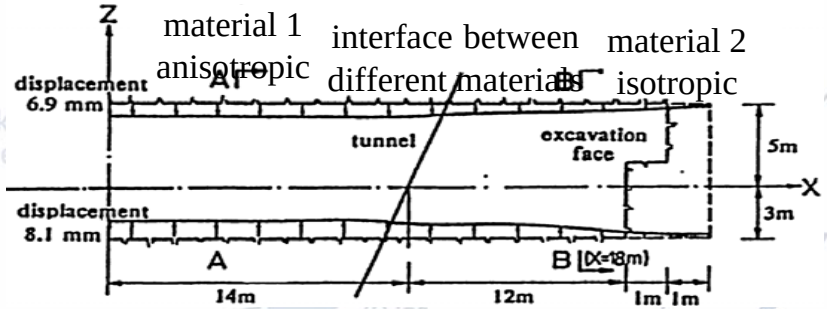
Publication: Z.S. Chen, G. Hofstetter, Z.K. Li, H. A. Mang, P. Torzicky, *Coupling of FE- and BE-Discretizations for 3D-Stress Analysis of Tunnels in Layered Anisotropic Rock.* Proceedings of the IUTAM/IACM Symposium on Discretization Methods in Structural Mechanics, G.Kuhn, H. Mang (Eds.), Springer-Verlag Berlin Heidelberg 1990.

# Computational mechanics in tunnel construction (2)

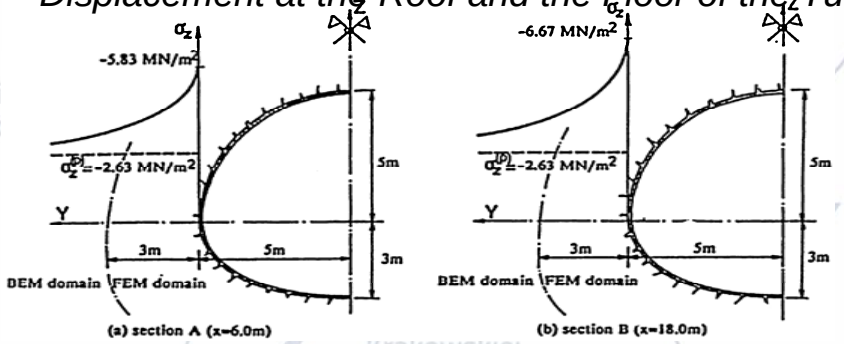
## Coupling of FE-and BE-Discretizations for 3D-Stress Analysis of Tunnels in Anisotropic Rock




*Typical Analysis Steps for the Simulation of the Excavation of the Tunnel*




*Displacement at the Roof and the Floor of the Tunnel*

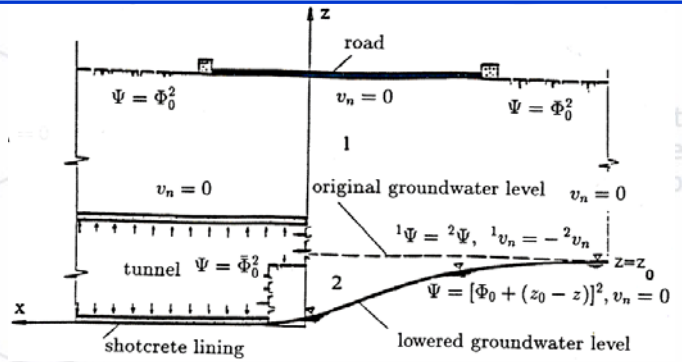



*Circumferential Stress along Lines x = 6.0m (18.0m), z = 0m*

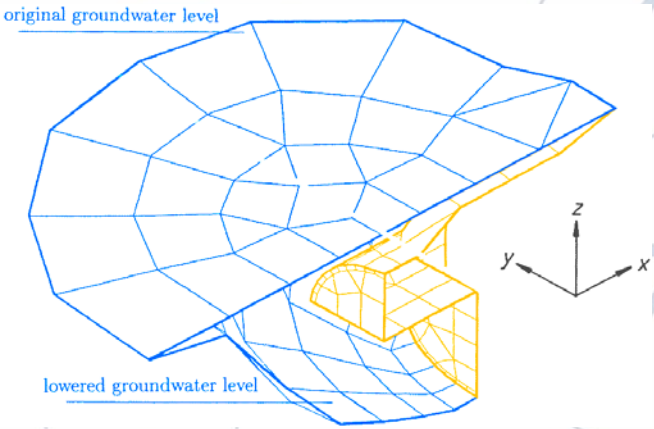
Publication: Z.S. Chen, G. Hofstetter, Z.K. Li, H. A. Mang, P. Torzicky, *Coupling of FE- and BE-Discretizations for 3D-Stress Analysis of Tunnels in Layered Anisotropic Rock.* Proceedings of the IUTAM/IACM Symposium on Discretization Methods in Structural Mechanics, G.Kuhn, H. Mang (Eds.), Springer-Verlag Berlin Heidelberg 1990.

# Computational mechanics in tunnel construction (3)

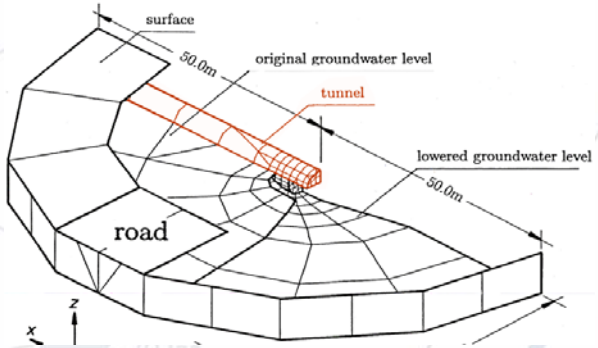
3D-Boundary Element Analysis of the Lowered Groundwater Table for Tunnels Driven Under Compressed Air



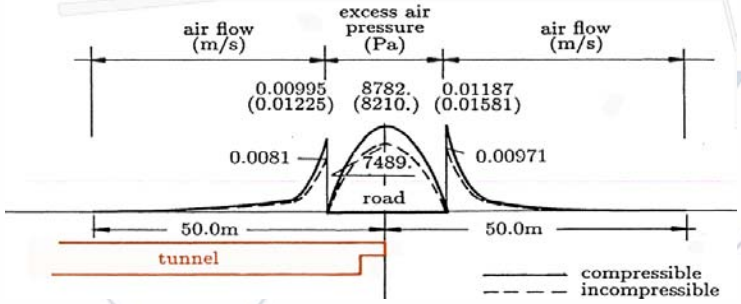

*Longitudinal View of a Tunnel Driven Under Compressed Air*




*Lowered Groundwater Level Obtained After Five Iteration Steps*




*Sketch of the Discretized Domain*

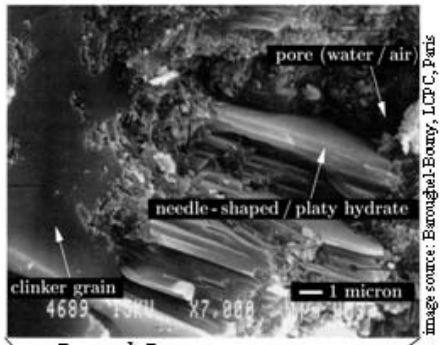



*Distribution of the Excess Air Pressure and the Air Flow*

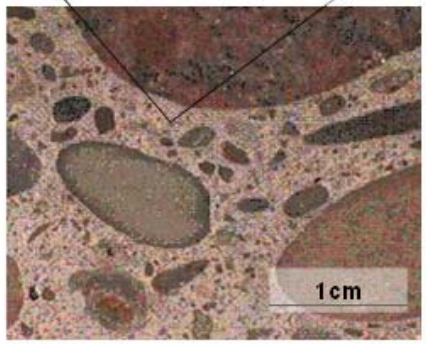
Publication: Z.S. Chen, G. Hofstetter, H.A. Mang, *3D-Boundary Element Analysis of the Lowered Groundwater Level for Tunnels Driven Under Compressed Air.* International Journal for Numerical and Analytical Methods in Geomechanics 15 (1991) 735-752

# Multiscale analysis of concrete and concrete structures (1)

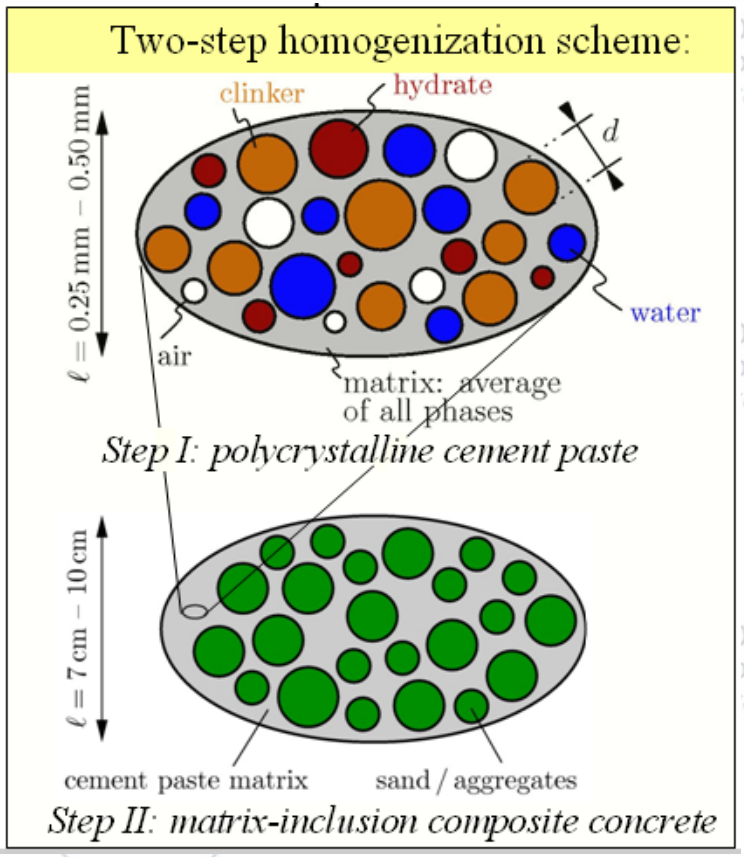
## Multiscale analysis of concrete




*Level I: cement paste*
clinker, water, hydrates, air




*Level II: concrete*
cement paste, aggregates (sand)




Publication: C. Hellmich, H.A. Mang, *Shotcrete Elasticity Revisited in the Framework of Continuum Micromechanics: From Submicron to Meter Level.* Journal of Materials in Civil Engineering 17 (2005) 433-447.

# Multiscale analysis of concrete and concrete structures (2)

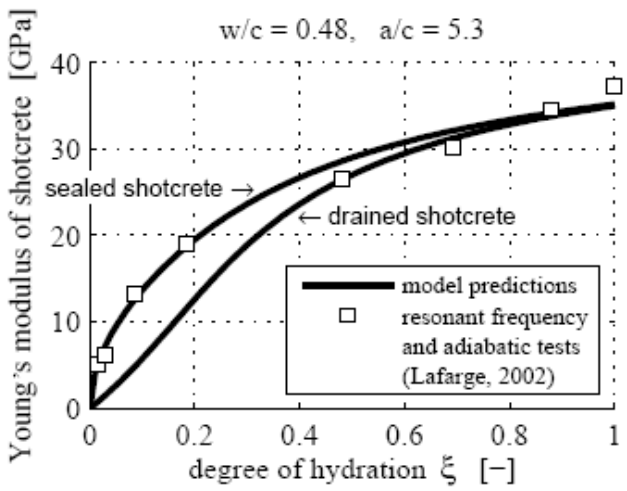
Elasticity homogenization of shotcrete: model inputs and model validation

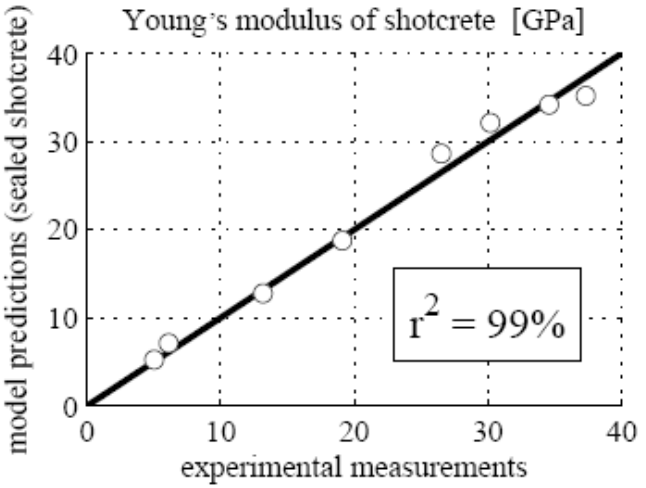
**Required input:**

1) elasticity constants of material phases

$k_{cement}$=116.7 GPa, $\mu_{cement}$=53.8GPa, $k_{water}$=2.3 GPa (sealed), $k_{water}$=0.0 GPa (drained), $\mu_{water}$=0.0 GPa, $k_{hydrates}$=14.1 GPa, $k_{aggregates}$=41.7 GPa, $\mu_{aggregates}$=19.2 GPa, $\mu_{hydrates}$=8.9 GPa

2) phase volume fractions $f_p$ determined through *Powers-type hydration model* (as functions of hydration degree, w/c, and a/c)




Excellent agreement between model predictions and experimental values.

# Multiscale analysis of concrete and concrete structures (3)

## Hybrid analysis of NATM tunnel shells: from measured displacements to strains

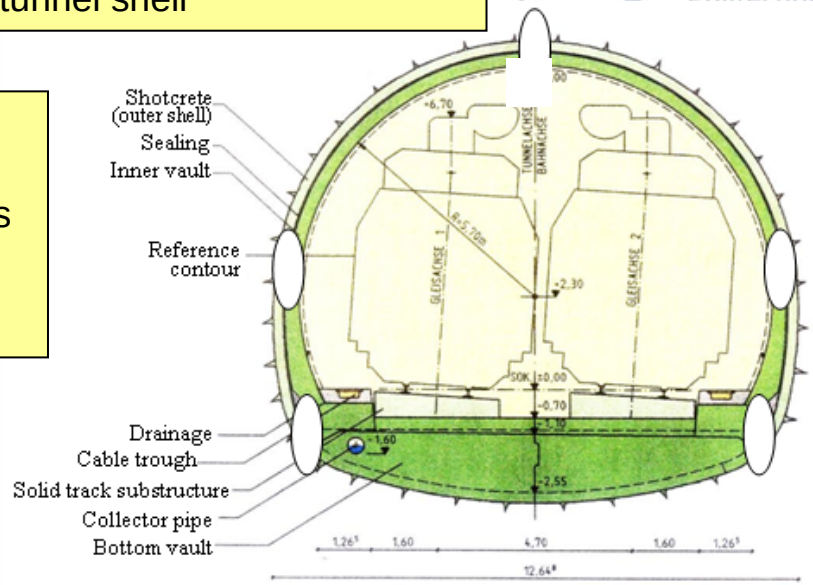
Every measurement cross-section is equipped with reflectors
daily laser-optical measurements of 3D displacement vectors

Transformation of measured displacement data
displacement components in moving base frame following the shell

Spatial and temporal interpolation of displacements
displacement field history of inner surface of tunnel shell

Kinematics of thin-shell theory
strain field history of entire tunnel shell,
serving as input for structural safety analyses

Benchmark example:
Sieberg tunnel (Austria)



# Multiscale analysis of concrete and concrete structures (4)

Hybrid analysis of NATM tunnel shells: from measured displacements to strains

Thermochemical analysis: solution of heat conduction problem considering exothermal chemical reaction

Field of hydration degree and temperature

Chemomechanical analysis: application of micromechanics-based material models (elasticity, creep, and strength)

Required inputs: fields of strain, hydration degree, and temperature

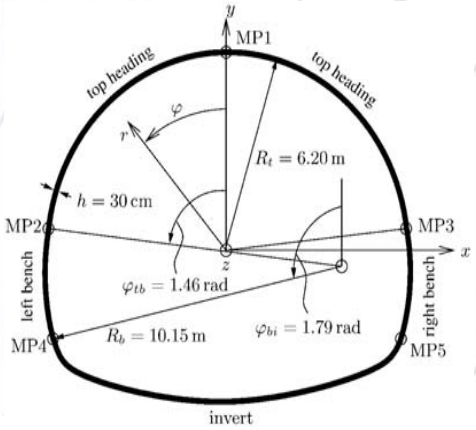
Computation of stresses **in** shotcrete tunnel shell

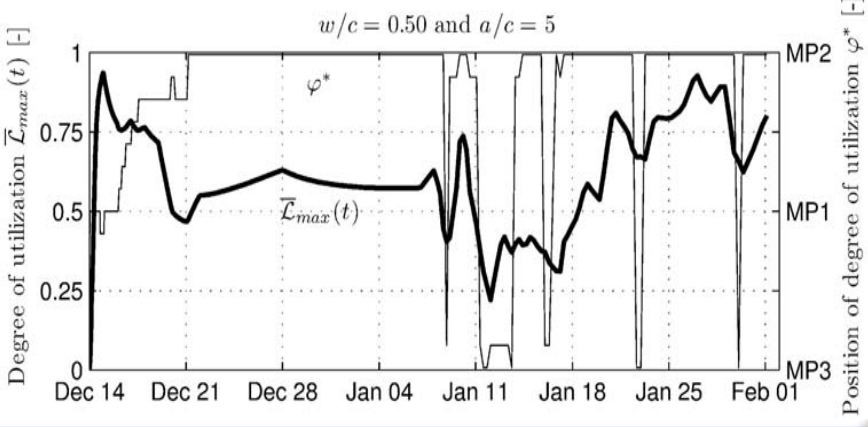
Comparison of stresses with ageing material strength

Evaluation of **degree of utilization**:

**0 … no loading of shell**

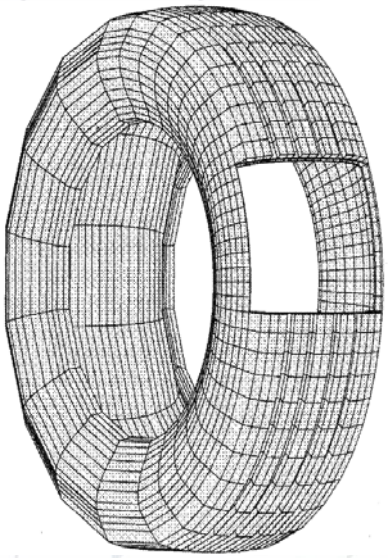
**1 … onset of failure**



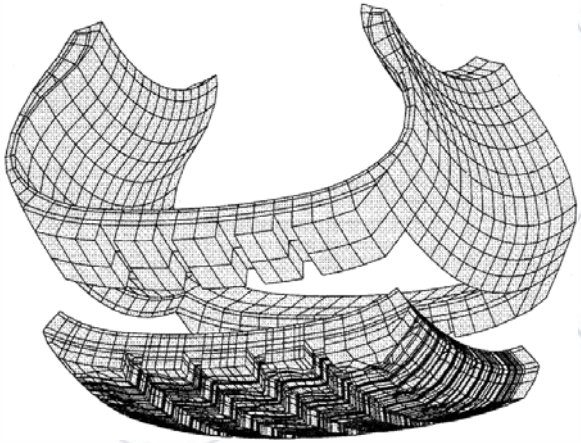



# Contact Mechanics

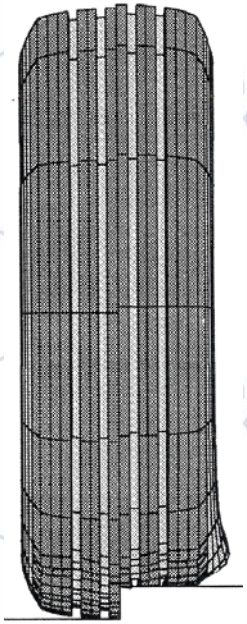
## 3D-FE Simulation of Automobile Tires



*Finite Element Mesh of the Tire Model with an Opening for the Refined*

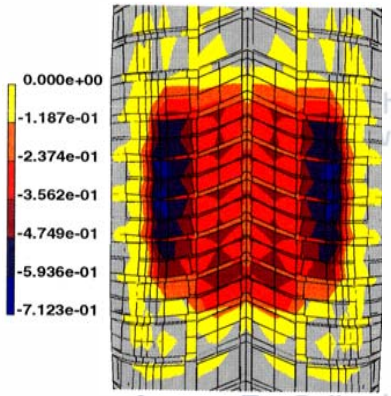


*Linking of the Above Mesh with a Refined Mesh*



*Undeformed and Deformed Tire Modell*

Simulation of the vertical load by incremental parallel movements of the rigid road surface towards the center of the tire




*Distribution of Contact Pressure After 15.7mm Frictional Sliding*

Publication: C.H. Liu, G.Meschke, P. Helnwein, H.A. Mang, *Tying Algorithm for Linking of Finite Element Meshes with Different Degrees of Refinement. Application to Finite Element Analyses of Tires.* Computer Assisted Mechanics and Engineering Sciences 2 (1995) 289-305.

# Scientific look at the future (1)

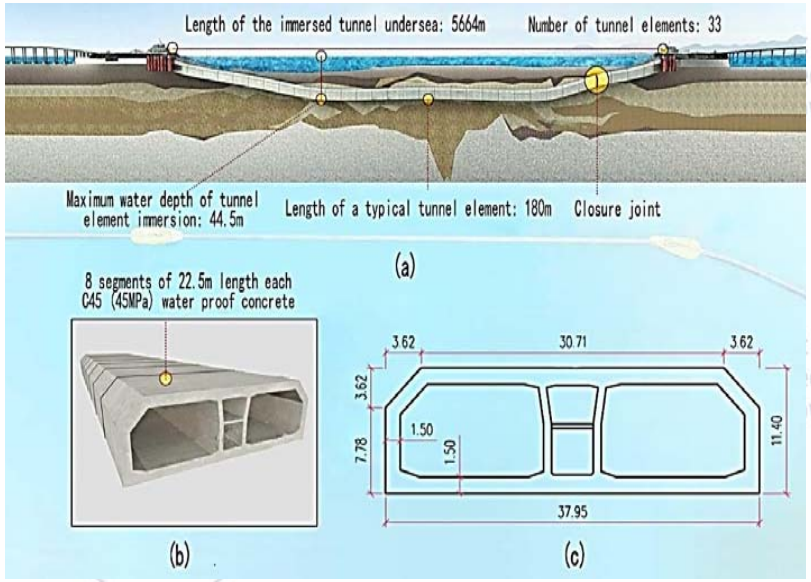
## Bridging the Gap

Connecting experimental testing with multiscale structural analyses to assess their added value

Joint research project of the Institute for Mechanics of Materials and Structures of Vienna University of Technology and the Department of Geotechnical Engineering of Tongji University, Shanghai.

Structure to be investigated: Submersed tunnel connecting the two parts of the Hong Kong-Zhuhai-Macao Bridge (HZMB)

*Course of the Hong Kong-Zhuhai-Macao Bridge*



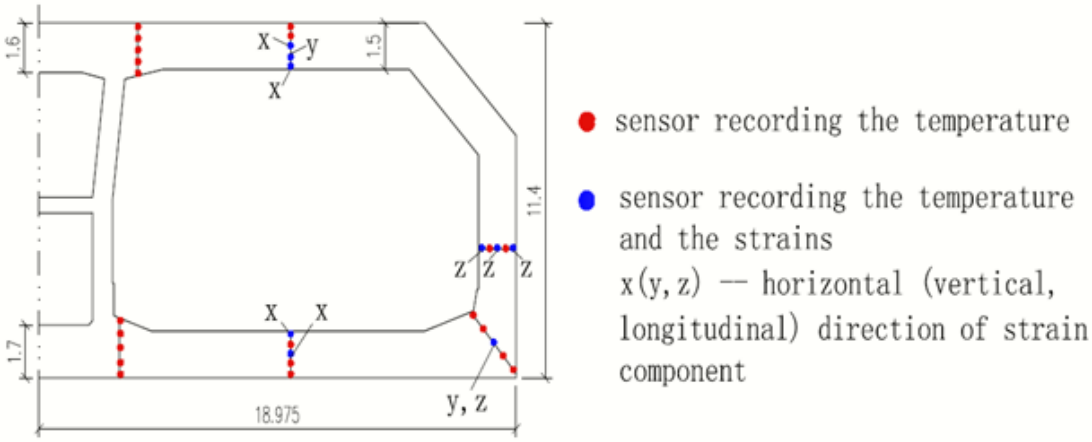

*Tunnel: (a) Longitudinal Section, (b) Typical Tunnel Element, (c) Cross-section of a Segment*

# Scientific look at the future (2)

## Bridging the Gap

| | |
|---|---|
| Task: | Assessment of the added value of multiscale analysis of the tunnel segments |
| Procedure: | Comparison of the results from 4 experimental tests with (a) results from conventional structural analyses and (b) results from multiscale analyses |

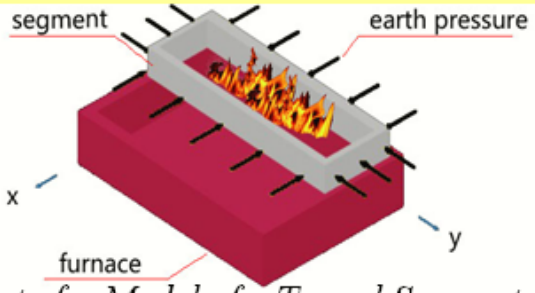
**Test #1**: Experimental determination of the early-age temperature and strain distribution in a precast segment of an element of the tunnel connecting the two parts of the HZMB




*One Half of Cross-section at Midspan of the Tested Tunnel Segment*

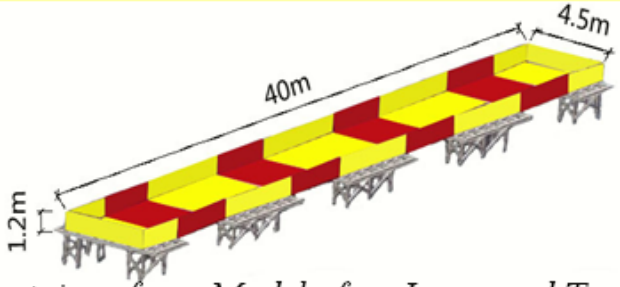
# Scientific look at the future (3)

**Test #2:** Experimental determination of the structural performance of large-scale models of a precast segment of an immersed tunnel, simultaneously subjected to bidirectional loading and fire



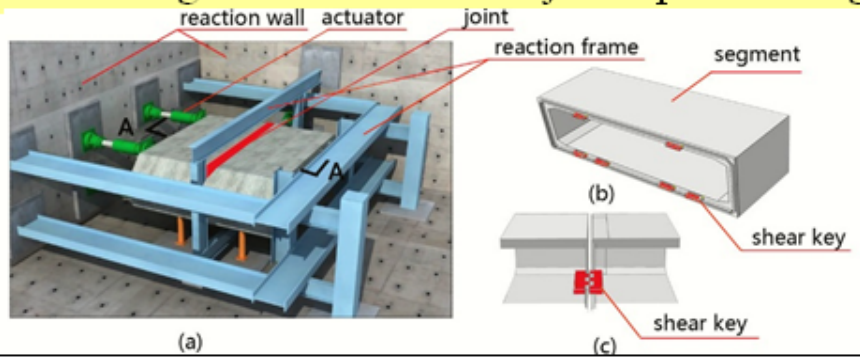

*Fire Test of a Model of a Tunnel Segment*

**Test #3:** Simulation of the dynamic behavior of models of a long portion of an immersed tunnel subjected to earthquake loads by means of multi-shaking-table tests




*Container for a Model of an Immersed Tunnel Subjected to a Multi-shaking-table Test*

**Test #4:** Static and dynamic experimental investigation of the mechanical behavior of the joints of large-scale models of adjacent parts of neighboring elements of an immersed tunnel



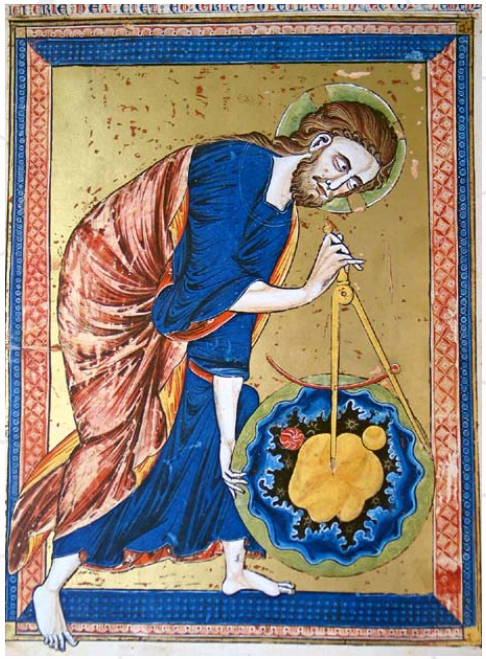

*Model of Adjacent Parts (Segments) of Neighboring Elements of an Immersed Tunnel:*
*(a) Test Set-up,*
*(b) Segment,*
*(c) Longitudinal Section A-A*

# Basic research as the condition for innovative solutions of challenging engineering problems (1)

## The Buckling Sphere - A Symbiosis of Mechanics and Geometry

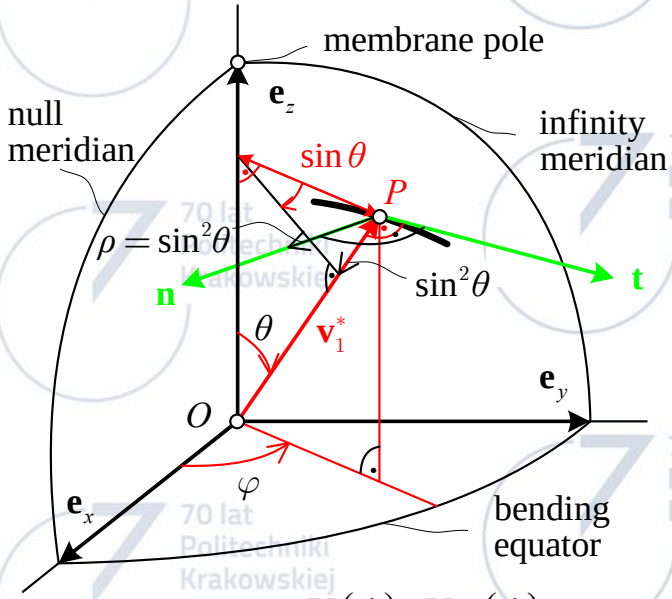
Intellectual foundation:

significance of geometry in the history of man



*From Bible Moralisée, France, around 1250*

Buckling sphere

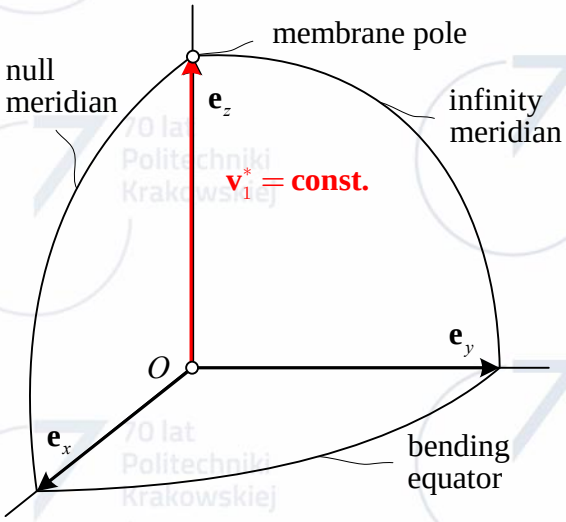



$$\rho(\lambda) = -\mathbf{v}_1^*(\lambda)\cdot\mathbf{n}(\lambda) = \frac{U(\lambda) - U_M(\lambda)}{U(\lambda)} = \sin^2\theta(\lambda)$$

$$\cos\varphi(\lambda) = \frac{\mathbf{v}_1^*(\lambda)\cdot\mathbf{v}_1^*(0) - \cos\theta(\lambda)\cos\theta(0)}{\sin\theta(\lambda)\sin\theta(0)}$$

# Basic research as the condition for innovative solutions of challenging engineering problems (2)

## The Buckling Sphere - A Symbiosis of Mechanics and Geometry



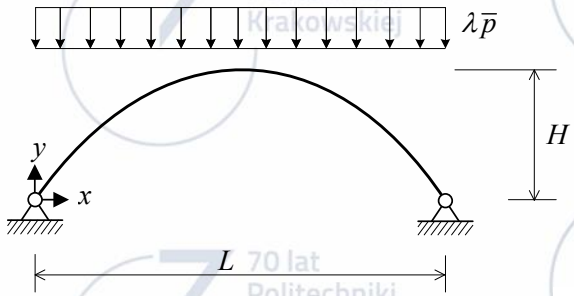

*Membrane stress state*

$$\mathbf{v}_1^*(\lambda) = \mathbf{const.} = \mathbf{v}_1$$

$$[\tilde{\mathbf{K}}_T + (\lambda_1^* - \lambda)\dot{\tilde{\mathbf{K}}}_T] \cdot \mathbf{v}_1^* = \mathbf{0}$$

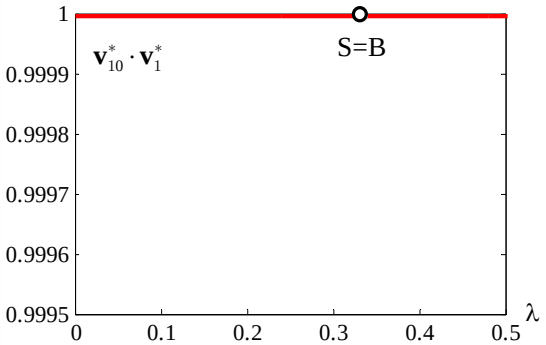
consistently linearized eigenproblem

$$\dot{\mathbf{v}}_1^* = \mathbf{0} \;\Rightarrow\; \mathbf{v}_j^* \cdot \ddot{\tilde{\mathbf{K}}}_T \cdot \mathbf{v}_1 = 0, \quad j = 2,3,...,N$$




$L = 600\ cm$
$H = 240\ cm$
$E = 20600\ kN/cm^2$
$A = 200\ cm^2$
$I = 6666.67\ cm^4$
$\bar{p} = 83.3\ kN/cm$

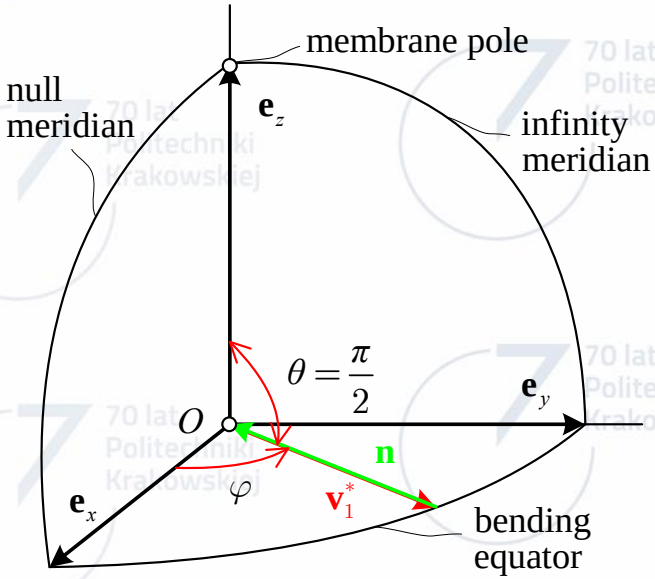
Two-hinged arch subjected to a vertical uniformly distributed load




$(\mathbf{v}_{10}^* \cdot \mathbf{v}_1^*)$ - $\lambda$ diagram $\Rightarrow$ Proof of $\mathbf{v}_1^* = \mathbf{const.}$

# Basic research as the condition for innovative solutions of challenging engineering problems (3)

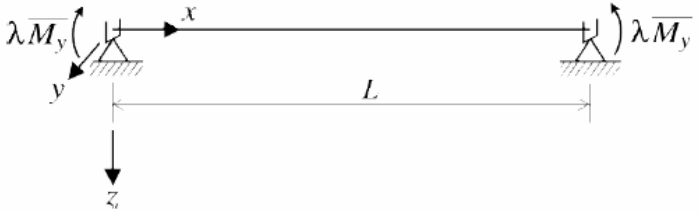
**The Buckling Sphere - A Symbiosis of Mechanics and Geometry**



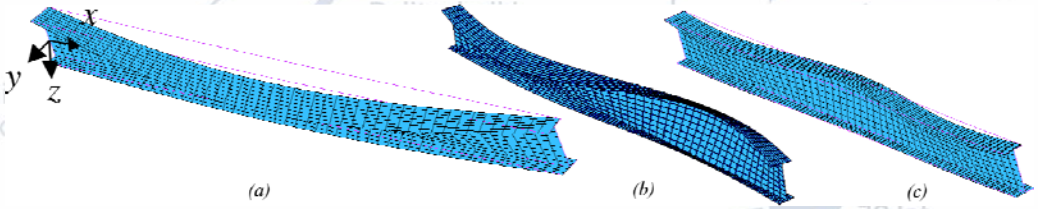

*Pure bending stress state*

$$\rho = 1 \Rightarrow \mathbf{n} = -\mathbf{v}_1^*$$

$$\mathbf{v}_1^* \cdot \mathbf{v}_j^* = 0, \quad j = 2,3,...,N$$




Fork-supported IPE-400 subjected to $\bar{M}_y$ at both ends




Lateral buckling of the above beam: first 3-eigenmodes

| | $j=2$ | $j=3$ | $j=4$ | $j=5$ |
|---|---|---|---|---|
| $v_j^* \cdot v_1^* = 0$ | $1.88 \cdot 10^{-10}$ | $4.16 \cdot 10^{-4}$ | $3.28 \cdot 10^{-10}$ | $4.50 \cdot 10^{-4}$ |

approximate satisfaction of the orthogonality conditions $\mathbf{v}_1^* \cdot \mathbf{v}_j^* = 0$, $j \neq 1$, e.g. for $j = 2,3,4,5$, at $\lambda$=0.

# Epilog

- Computational mechanics is indeed an ideal research area for innovative solutions of challenging engineering problems.
- The personal involvement in the development of computational mechanics for nearly 50 years is considered as a privilege.
- My personal relations to Cracow University of Technology, where computational mechanics plays a prominent role, is also regarded as a privilege.

Congratulations to CUT on her 70th Anniversary and best wishes for sustained scientific success in the future!